On Modeling of Earth Rotation Variations in an Integrated Model and Appropriate Frames of Reference

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Summary

We consider the modular approach to the modeling of Earth's rotation developed recently and discuss some improvements of the model. In this approach, the Earth is described by a number of mutually interacting subsystems which all have their own dynamics. The subsystems geometrically represent distinct layers of the Earth. As a result of their dynamics and interaction, these layers deform and move relative to one another. This situation raises the question what frames of reference are appropriate for the description of the distinct subsystems. We argue that each subsystem should be described in its own 'internal' frame of reference adapted to the needs of its particular physics. In addition there must be a global frame of reference for the whole system in which Earth's rotation as an integral quantity of the whole system is defined and which also serves for the description of interactions of distinct subsystems.

1 Introduction

Modeling of the Earth focusing on mechanical properties of the geosphere generally attempts to describe the whole Earth by a single system of equations specialized for specific phenomena (see e.g. LAMBECK, 1988; WAHR, 1981 for rotation and loading deformations, respectively). In order to be feasible, this approach requires a high degree of simplification and many interactions and feedbacks have to be neglected. Consequently, even the most advanced geophysical models presently available are highly simplified and, moreover, specialized for the description of specific phenomena (such as nutation, Earth tides, surface deformations, geoid anomalies, glacial loading).

Over the past decades, the development of geophysical and geodetic observations techniques (in particular, space-geodetic methods such as VLBI, SLR and GPS and absolute and relative gravimetry) has pushed observational accuracies far beyond the limits of these geophysical models. Moreover, the most advanced observation techniques are increasingly applied to the study of global change phenomena, where specific contributions to mass movements or changes in the dynamics of the system need to be isolated in the observations (for examples, see WAHR AND HAN, 1997; PELTIER AND JIANG, 1997).

The accuracy of the isolated signal strongly depends on the quality of the geophysical models used to correct for known effects in the observations. Thus, the status of the geophysical models is a limiting factor in the understanding of the observation, the isolation of relevant signals, and the improvement of the observational techniques themselves (for a comprehensive discussion, see PLAG *et al.*, 1998).

Recently, Jüttner and Plag (1998) have developed a modular approach to the modeling of Earth's rotation. In this modular approach to the dynamics of the rotating Earth, the Earth is represented by a number of physically defined subsystems coupled to each other both by boundary conditions and far-field interactions. Thereby, different subsystems are described each on its own by dynamical equations.

Couplings between different subsystems in this approach are defined as physically meaningful quantities, e.g. forces, moments or fields. Thus, they are independent of the particular structure and dynamics of the subsystems. There might be, moreover, external excitations acting on one or several particular subsystems such as for instance a tidal potential.

Unlike a uniform description of the rotating planet as a whole, the modular approach to planetary dynamics requires direct time domain integration (see JÜTTNER AND PLAG, 1998). In the modular approach, the system is integrated as an initial value problem.

Such a modular approach allows for successive sophistications individually inside each of the subsystems without requiring any changes in the other subsystems. The only demands on the mathematical description of dynamics of a subsystem are that it has to supply the other subsystems with time-dependent values of the prescribed physical coupling parameters and that it has to work with such coupling parameters supplied to it by other subsystems. Of course, special attention has to be paid to the definition of the subsystems and their interactions right at the outset. Indeed, the isolation of subsystems of the planet and convention on the kind of their mutual interactions defines the structure of the modular theory.

2 Development of the model

The validity and feasibility of the modular approach has been demonstrated by a simple model based on Euler-Liouville equations for mantle and core (see JÜTTNER AND PLAG, 1998). The structure of this first version of a ***D****ynamical* ***I****ntegrated* ***M****odular* ***E****arth* ***R****otation* ***S****ystem* (DIMERS-V1) is shown in figure 1 below. The model consists of two quasi rigidly rotating shells, core and mantle. The crust is included in the

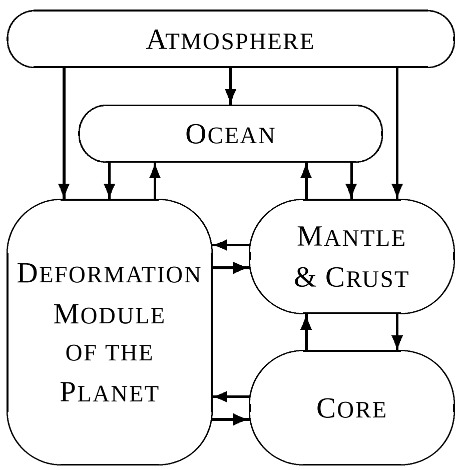

Figure 1: Subsystems and interactions of the initial version of the system model DIMERS-V1. The boxes represent the distinct subsystems and modules and an arrow indicates an action of a subsystem on the subsystem to which it points.

Table 1: Periods in sidereal days and quality factors of the Chandler wobble calculated with DIMERS-V1 for different ocean models and a typical choice of parameters.

ocean	T_{CW}	Q_{CW}
none	411.862 d	67.2226
global	454.725 d	60.8974
ETOPO5	441.447 d	62.7257

mantle subsystem. An atmosphere and an ocean have been added to the model. The atmosphere subsystem represents measured or synthetic pressure data. The ocean has been taken to be in equilibrium with atmospheric pressure, varying rotational and gravitational potential and sea floor elevations. Its coast lines have been extracted from the data set ETOPO5.

Deformations have been treated on the basis of static Love numbers for a spherically symmetric and non-rotating planet. This treatment of deformations deviating from the concept of subsystems with self-contained dynamics, however, has forced the addition of an artificial deformation module to DIMERS-V1. It does not represent a physically defined subsystem and the deformation of the physical subsystems appear as artificial interactions with the deformation module.

As may be expected from the simple structure of subsystems of DIMERS-V1, this model shows two rotational eigenmodes, the Chandler wobble and the nearly diurnal free wobble. The period of the Chandler wobble in this model is about 441 sidereal days depending slightly on the coupling parameters chosen. This period, however, is seriously influenced by the ocean model as may be seen from table 1 (table 3 of Jüttner and Plag, 1998).

The model is currently being improved to include a solid inner and a fluid outer core. Now, deformations are treated dynamically within any deformable subsystem separately. This second version DIMERS-V2 of the model also has a module representing the tidal potential. Figure 2 shows the structure of DIMERS-V2. While DIMERS-V1 is based on the balance of the total angular momentums of the subsystems, DIMERS-V2 is defined by continuum mechanical equations of motion within mantle, inner and outer core.

The inner core and the subsystem of mantle and crust, for brevity henceforth called 'mantle', are now going to be described in Lagrangian coordinates by the theory of viscoelasticity. If the rheological models characterizing the viscoelastic solids of mantle and inner

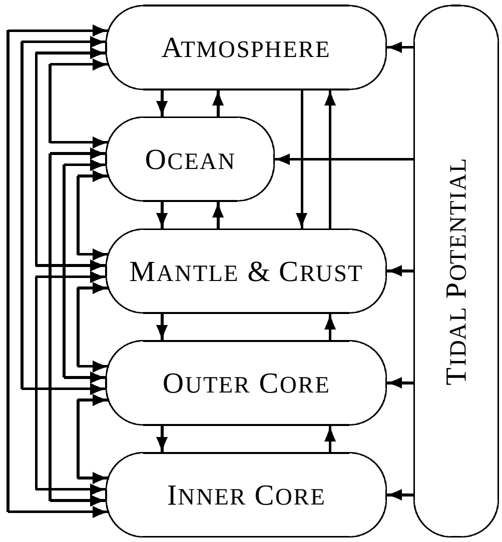

Figure 2: Structure of the integrated system model DIMERS-V2 as in figure 1. Vertical arrows between boxes represent interaction processes at the common boundary of the subsystems joined by them. Arrows going horizontally into the boxes represent remote actions by fields defined in the bulk of the subsystems.

core have only a finite number of relaxation times, like the Maxwell or the Burgers rheology, there is no difficulty with time domain integration as an initial value problem. In this case, the stress tensor is determined by a differential equation in the time variable which may be integrated along with the differential equations governing the dynamics.

However, if a rheological model has a continuum of relaxation times like the Zschau rheology, the stress tensor is determined by an integro-differential equation. That would require significant changes in the integration procedure of the whole system model. For this reason, the rheological models dealt with in DIMERS-V2 are restricted to a finite number of relaxation times and a continuum of relaxation times in the context of the modular approach must be a matter of independent research.

The outer core shall be treated hydrodynamically in Eulerian coordinates or, on a later stage, magnetohydrodynamically. The inner core may move and rotate relative to the mantle. Moreover, the electrical conductivity of the inner core has to be taken into account if the outer core is treated magnetohydrodynamically.

These three subsystems of DIMERS-V2 are geometrically defined as slightly elliptical shells with an ellipticity varying with radius. In the theoretical description, this is reduced to spherical shells and a function of radius in each shell which describes the ellipticity

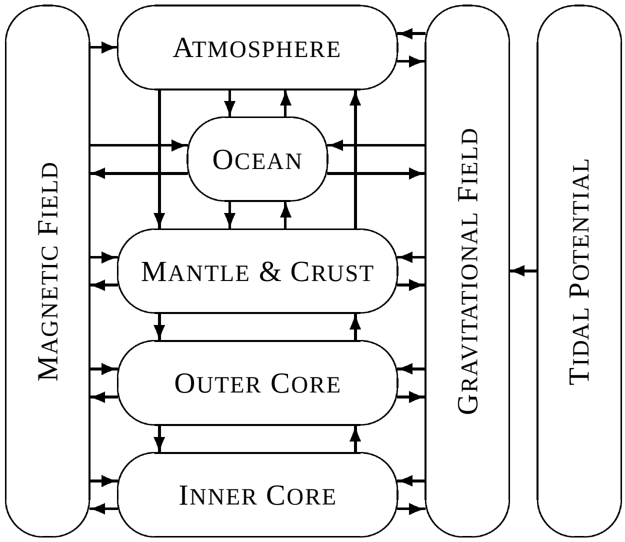

Figure 3: Structure of the integrated system model DIMERS-V2 as in figure 2 but with far-field interactions integrated into field modules.

of that shell, i.e. these shells are regarded as deformed spherical shells.

In DIMERS-V2, the ocean subsystem will be defined as in DIMERS-V1. The atmosphere subsystem shall be extended to represent a particular atmospheric dynamics. Time series of atmospheric pressure and wind velocities from several different heights shall be assimilated with this mechanical model of the atmosphere. The new module of the tidal potential will provide the tidal potential on the basis of ephemeries.

Under such circumstances, scalar, vector and tensor fields defined within these shells can be approximated by truncated series of spherical harmonic functions with coefficients being polynomials in the radius. Such fields, e.g. the displacement vector and the stress tensor in mantle and inner core, the velocity vector and dynamical pressure in the outer core or the gravitational field in every shell, are part of the dynamical variables to be integrated by DIMERS-V2.

Mathematical operations on these fields, like, e.g., multiplication by a scalar field, vector product, gradient of divergence, are made available to the program independent of the physical meaning of the fields and of the shell where they are defined. Furthermore, general integration routines are developed for differential equations in such fields.

With these general tools at hand, the programming of dynamics of the subsystems follows more or less directly the abstract formulation of the differential equations governing the dynamics. Thereby, the development of dynamical models for the subsystems and their integration into the modular system model is considerably simplified.

The structure of the system model is conceptually simplified if far-field interactions are integrated into field modules representing physical fields. Figure 3 shows the structure of DIMERS-V2 with field interactions integrated this way. A gravitational and a magnetic field are displayed. The only function of these two modules is to sum all field contributions from the physical subsystems for the gravitational or magnetic field respectively.

3 Frames of reference in a modular integrated model

A further concern relates to the frame of reference chosen to describe the motions and fields within each of the subsystems. It is compelling to have a single frame of reference for the description of all distinct subsystems. A description with different frames of reference for different subsystems would necessitate coordinate transforms between the different frames of reference in calculating any interaction process. Therefore, in this case, any subsystem must know the frames of reference of all its interaction partners which contradicts the modular approach.

On the other hand, especially a description in Lagrangian coordinates, as intended for mantle and inner core, requires a frame of reference in which all material points do not move far away from their original position. In principal, the strain tensor is not affected if a frame of reference does not meet this requirement. Then, however, a small strain tensor is calculated from a very large displacement field which leads to numerical instabilities. As the relative position and orientation of mantle and inner core changes with time, no single frame of reference can meet this requirement for both subsystems.

The solution of this contradiction is to have one ‘global’ frame of reference for the whole system model and, in addition, ‘internal’ frames of reference deviating from the global one in some of the subsystems. Then, the modules themselves are responsible for the transformation from their internal frame of reference to the global one. In general, this transformation will be determined by a differential equation, as well. It has to be integrated along with the dynamics of the subsystem.

From the point of view of dynamics, for a description in Lagrangian coordinates, the 'dynamical Tisserand system' of a subsystem with origin placed in the center of mass of that subsystem is best suited as internal frame of reference. It is defined by the property that the total angular momentum just as the total linear momentum of the motion of all mass elements of the subsystem relative to this frame of reference vanishes. Thereby, the mean velocity square of all material points of the subsystem weighted by the mass is minimized in this frame of reference. A different frame of reference satisfying the condition that material points remain close to their original position might, however, replace the dynamical Tisserand system, if other arguments are in favour of it.

The global frame of reference, however, should be chosen from the point of view of comparism of the simulations with observations. Since observations of Earth rotation are based on stations on the surface of the Earth, this might reasonably be done by defining the global frame of reference on the basis of surface kinematics, for instance by means of the mean surface vorticity.

4 Conclusions

The modular approach to Earth rotation dynamics developed in an earlier paper (Jüttner and Plag, 1998) represents a novel way of thinking about this problem. The validity and feasibility of the approach was demonstrated in that paper for a first simplified model of the mechanical Earth system.

The model is now being improved to have a complex, more realistic structure. In the new version DIMERS-V2, it comprises the subsystems of an inner and an outer core, a mantle, an ocean and an atmosphere. The action of the tidal potential is represented by an additional module. The dynamics of the individual subsystems are described by partial differential equations for scalar, vector and tensor fields.

Such a modular model requires different frames of reference in the description of the distinct subsystems which are called 'internal' to their respective subsystem. Especially, in the case of a description in Lagrangian coordinates, the internal frame of reference has to be adapted so that all material points remain close to their original position.

Of course, the description of the modular model with several different frames of reference internal to the subsystems has to be supplemented by a 'global' frame of reference in which integral quantities of the whole system are defined. Furthermore, this global frame of reference is a prerequisite for an unambiguous definition of subsystem interactions. Coordinate transforms have to be supplied between any internal frame of reference and the global one but not for all the pairs of distinct internal frames of reference.

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